

Nernst Effect as a Signature of Quantum Fluctuations in Quasi-1D Superconductors

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We study a model for the transverse thermoelectric response due to quantum superconducting fluctuations in a two-leg Josephson ladder, subject to a perpendicular magnetic field B and a transverse temperature gradient. The off-diagonal Peltier coefficient (α_{xy}) and the Nernst effect are evaluated as functions of B and the temperature T . The Nernst effect is found to exhibit a prominent peak close to the superconductor–insulator transition (SIT), which becomes progressively enhanced at low T . In addition, we derive a relation to diamagnetic response: $\alpha_{xy} = -M/T_0$, where M is the equilibrium magnetization and T_0 a plasma energy in the superconducting legs.

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In low-dimensional superconducting (SC) systems (thin films, wires and Josephson arrays), fluctuations of the SC order parameter field lead to broadening of the transition to the SC state, and give rise to anomalous transport properties in the adjacent normal phase [1]. While close to or above the critical temperature T_c thermally excited fluctuations dominate these conduction anomalies, quantum fluctuations are expected to dominate at low temperatures $T \ll T_c$, where superconductivity is weakened due to, e.g., the effect of a magnetic field, disorder or repulsive Coulomb interactions. Their most dramatic manifestation is the onset of a superconductor–insulator transition (SIT) when an external parameter such as magnetic field or thickness is tuned beyond a critical point [2, 3].

A striking signature of the fluctuations regime, which attracted much attention in the recent years, is the anomalous enhancement of transverse thermoelectric effects in the presence of a perpendicular magnetic field B . In particular, a substantial Nernst effect measured far above T_c , e.g., in the underdoped regime of high- T_c superconductors [4, 5] and disordered thin films [6]. As the Nernst signal (a voltage developing in response to a temperature gradient in the perpendicular direction) is typically small in ordinary metals, its magnification in such systems has been attributed to the dynamics of thermally excited Gaussian SC fluctuations [7–9], or mobile vortices above a Kosterlitz-Thouless [10] transition [11]. Theoretical studies have also been extended to the quantum critical regime of SC fluctuations [12].

Conceptually, the above mentioned theoretical models share a common intuitive idea: in the phase-disordered, vortex liquid state, vortex flow generated parallel to a thermal gradient ($\nabla_y T$) naturally induces an electric field (E_x) in the perpendicular direction. Consequently, the general expression for the Nernst coefficient

$$\nu \equiv \frac{E_x}{(\nabla_y T)B} = \frac{\rho_{xx}\alpha_{xy} - \rho_{xy}\alpha_{yy}}{B} \quad (1)$$

is overwhelmingly dominated by the first term, dictated by the off-diagonal Peltier coefficient α_{xy} . The latter is

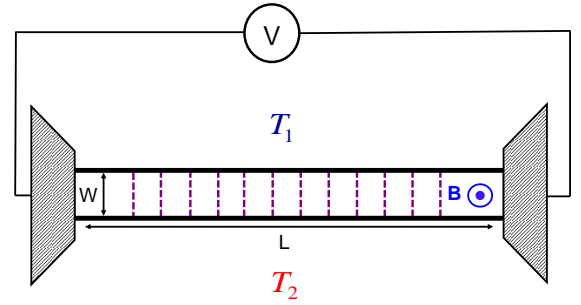


FIG. 1: (color online) A scheme for measurement of the Nernst effect in a Josephson ladder subject to a magnetic field B perpendicular to the plane, and a temperature difference between the top (T_1) and bottom (T_2) SC wires. Dashed lines represent Josephson coupling.

an interesting quantity: a transport coefficient, yet intimately related to thermodynamic properties. In particular, it was found to be proportional to the diamagnetic response [5, 8, 11]. In the clean limit, it was shown to encode the entropy per carrier [13, 14]. Note, however, that the implied contribution to ν [Eq. (1)] is not determined by α_{xy} alone, but rather its product with the electric resistivity ρ_{xx} . Observation of a large Nernst signal therefore necessitates a reasonably resistive normal state.

In this paper, we present a theory for the transverse thermoelectric coefficients and their relation to diamagnetism in the quasi one-dimensional (1D) superconducting device depicted in Fig. 1, in which the geometry dictates an appreciable Nernst effect in the fluctuations-dominated regime. The device considered is a two-leg Josephson ladder subject to a perpendicular magnetic field B , where a small temperature difference between the legs induces voltage along the ladder due to transport of vortices across the junction. At low T , one expects vortex transport to be dominated by quantum tunneling. This

system serves as a minimal setup for observing transverse thermoelectric effects; the relative simplicity of the model describing the quantum dynamics of SC fluctuations allows an explicit evaluation of α_{xy} , ν and the magnetization density M in a wide range of parameters. In particular, we investigate their behavior when the wires parameters are tuned through a SIT, and find a prominent peak in ν close to the transition, which becomes progressively enhanced at low T . We further confirm the proportionality relation between α_{xy} and $-M$, however the prefactor is $1/T_0$, with T_0 the plasma energy scale, rather than $1/T$ as found in the 2D case [8, 11].

As indicated in Fig. 1, the system consists of two SC wires of length L parallel to the x direction separated by a thin insulator layer of width W , which allows a weak Josephson coupling J per unit length. In each of the separate wires ($n = 1, 2$), the 1D quantum dynamics of fluctuations in the phase of the SC condensate is governed by a Hamiltonian of the form (in units where $\hbar = 1$)

$$H_n = \frac{v}{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[K(\partial_x \theta_n)^2 + \frac{1}{K}(\partial_x \phi_n)^2 \right]; \quad (2)$$

here $\phi_n(x)$ is the collective phase field, and $\theta_n(x)$ its conjugate field (obeying $[\phi_n(x), \partial_x \theta_n(x')] = i\pi\delta(x' - x)$) which denotes Cooper pair number fluctuations. This model can be viewed as describing, e.g., the continuum limit of a Josephson chain [3], where the Josephson coupling (E_J) and charging energy (E_c) between adjacent SC grains are related to the parameters of H_n by $K = \sqrt{E_c/E_J}$ and $v = \sqrt{E_c E_J} \pi a$, with a the grain size. The Josephson coupling between the wires is given by

$$H_J = -J \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \cos\{\phi_1 - \phi_2 - qx\} \quad (3)$$

where q is the deviation of the vortex density in the junction area from a commensurate value:

$$q = 2\pi \left(\frac{WB}{\Phi_0} \bmod \frac{1}{a} \right) \quad (4)$$

in which (for $\hbar = c = 1$) $\Phi_0 = \pi/e$ is the flux quantum.

Assuming the hierarchy of scales $Ja \ll T \ll v/a$ (with T an average temperature of the system), H_J [Eq. (3)] can be treated perturbatively. Note that the first inequality justifies this approximation for an arbitrary value of the Luttinger parameter K in Eq. (2): for $K < 2$ and

sufficiently small q , the Josephson term becomes relevant [15], and induces a SC phase where fluctuations in the relative phase between the wires are gapped [16, 17] in the $T \rightarrow 0$ limit. In turn, additional perturbations such as inter-wire charging energy [16, 17] and disorder [18] generate a transition to an insulating $T \rightarrow 0$ phase for sufficiently large K . Since, as shown below, in both extreme phases the Nernst effect is strongly suppressed, we focus our attention on the intermediate T regime. In addition to H_J , we introduce a weak backscattering term due to disorder of the form

$$H_D = \sum_{n=1,2} \int dx \zeta_n(x) \cos\{2\theta_n(x)\}, \quad (5)$$

$$\overline{\zeta_n(x)} = 0, \quad \overline{\zeta_n(x)\zeta_{n'}(x')} = D\delta(x-x')\delta_{n,n'}$$

where overline denotes disorder averaging. This term generates the leading contribution to the resistivity ρ_{xx} , and thus to ν via Eq. (1).

We now consider a temperature difference $\Delta T = T_1 - T_2$ between the top and bottom wires (see Fig. 1), each assumed to be at equilibrium with a separate heat reservoir, and focus first on the transverse Peltier coefficient α_{xy} . Evaluating the electric current I_x induced along the ladder for $\Delta T \ll T$ yields $\alpha_{xy} = I_x/\Delta T$. Alternatively, $\tilde{\alpha}_{xy} = T\alpha_{xy}$ can be derived from the heat current $I_x^{(h)}$ generated by a voltage difference V_y between the wires. We show below that the result of both calculations is the same, as dictated by the Onsager relation [8].

The electric current is given by the expectation value

$$I_x = 2e\pi \langle \dot{\theta}_1 + \dot{\theta}_2 \rangle \quad (6)$$

where the current operators $\dot{\theta}_n(x, t) = i[H, \theta_n]$ ($H = H_0 + H_J$ where $H_0 \equiv H_1 + H_2$) are evaluated perturbatively in H_J using the interaction representation. The leading contribution to I_x arises from the second order:

$$\dot{\theta}_n(x, t) = U(t)\dot{\theta}_n^{(0)}(x, t)U^\dagger(t) \quad \text{where}$$

$$\dot{\theta}_n^{(0)}(x, t) = \frac{v}{K} \partial_x \phi_n(x, t), \quad (7)$$

$$U(t) = 1 + i \int_{-\infty}^t dt_1 H_J(t_1) - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 H_J(t_1) H_J(t_2).$$

Employing Eq. (3) and inserting the resulting expressions for θ_n in Eq. (6), we obtain

$$I_x = \frac{\pi e v J^2}{2} \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_1 \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_2 \sin[q(x_1 - x_2)] i \Im m \left[e^{-\frac{K}{2} F_{T_1}(x_1 - x_2; t_1 - t_2)} e^{-\frac{K}{2} F_{T_2}(x_1 - x_2; t_1 - t_2)} \right] \quad (8)$$

$$[\{\partial_x F_{T_1}(x - x_1; t - t_1) + \partial_x F_{T_1}(x_2 - x; t_2 - t)\} - \{\partial_x F_{T_2}(x - x_1; t - t_1) + \partial_x F_{T_2}(x_2 - x; t_2 - t)\} \\ - \{\partial_x F_{T_1}(x - x_2; t - t_1) + \partial_x F_{T_1}(x_1 - x; t_2 - t)\} + \{\partial_x F_{T_2}(x - x_2; t - t_1) + \partial_x F_{T_2}(x_1 - x; t_2 - t)\}] ,$$

where we use the Boson correlation function $F_T(x; t) \equiv \frac{1}{K} \langle [\phi_n(x, t) - \phi_n(0, 0)]^2 \rangle$ at fixed temperature T [15]:

$$F_T(x; t) = \frac{1}{2} \log \left[\frac{v^2}{\pi^2 a^2 T^2} \left\{ \sinh \left[\pi T \left(\frac{x}{v} - t + i \frac{a}{v} \text{sign}(t) \right) \right] \sinh \left[\pi T \left(\frac{x}{v} + t - i \frac{a}{v} \text{sign}(t) \right) \right] \right\} \right]. \quad (9)$$

For a small temperature difference between the wires ($\Delta T \ll T = \frac{T_1 + T_2}{2}$) and $L \rightarrow \infty$, this yields

$$\alpha_{xy} = \frac{I_x}{\Delta T} = -\frac{e(\pi J)^2 a^3}{4v^2} \sin \left(\frac{\pi K}{2} \right) \left(\frac{2\pi a T}{v} \right)^{K-2} \partial_q \left\{ B \left(-i \frac{vq}{4\pi T} + \frac{K}{4}, 1 - \frac{K}{2} \right) B \left(i \frac{vq}{4\pi T} + \frac{K}{4}, 1 - \frac{K}{2} \right) \right\} \quad (10)$$

where $B(z, w)$ is the Beta function.

We next consider the alternative setup where an electric voltage V_y is imposed between the top and bottom wires (at uniform T), and evaluate the heat current $I_x^{(h)}$ induced along the ladder. For $J = 0$, the local heat current operator is given by [19]

$$J_h^{(0)} = v^2 \sum_{n=1,2} \partial_x \phi_n \partial_x \theta_n. \quad (11)$$

The voltage bias corresponds to a difference in chemical potentials in the two legs, $\mu_{1,2} = \pm eV_y$, which introduce constant shifts of $\partial_x \theta_{1,2}$ by $\pm \pi eV_y / vK$. The heat current $I_x^{(h)} = \langle U(t) J_h^{(0)} U^\dagger(t) \rangle$ [with $U(t)$ expanded to second order in H_J as in Eq. (7)], is hence given by

$$I_x^{(h)} = \frac{e\pi v V_y}{K} \langle U(t) (\partial_x \phi_1 - \partial_x \phi_2) U^\dagger(t) \rangle. \quad (12)$$

The resulting expression coincides with $(V_y T / \Delta T) I_x$, with I_x given by Eq. (8) for $\Delta T \ll T$. We thus confirm that $\tilde{\alpha}_{xy} = I_x^{(h)} / V_y = T \alpha_{xy}$.

To derive the Nernst coefficient, we employ Eq. (1) noting that within our level of approximations, α_{yy} and ρ_{xy} vanish due to particle-hole symmetry. The Nernst signal in the setup depicted in Fig. 1, defined as $\nu = V / \Delta T B$, is hence determined by the product of α_{xy} [Eq. (10)] and the longitudinal resistance of the ladder R_{xx} . To leading order in H_D [Eq. (5)] [3, 15, 18],

$$R_{xx} = \frac{\pi^3 L D a^2}{2e^2 v^2} \cos \left(\frac{\pi}{K} \right) B \left(\frac{1}{K}, 1 - \frac{2}{K} \right) \left(\frac{2\pi a T}{v} \right)^{\frac{2}{K}-2}. \quad (13)$$

At low magnetic field such that $q = 2\pi W B / \Phi_0 \ll T/v$, this yields an expression for $\nu \approx \alpha_{xy} R_{xx} / B$ of the form

$$\nu \approx \nu_0 \mathcal{F}(K) \left(\frac{2\pi a T}{v} \right)^{K + \frac{2}{K} - 6} \quad (14)$$

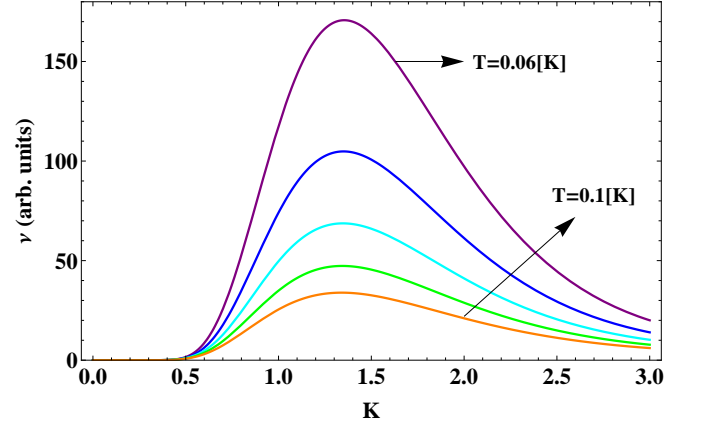


FIG. 2: (color online) Isotherms of ν as a function of K for $v/2\pi a = 1K$, $T = 0.06, 0.07, 0.08, 0.09, 0.1K$.

where the constant prefactor $\nu_0 \propto L D J^2$ and

$$\mathcal{F}(K) \equiv \frac{\Gamma^2 \left(\frac{K}{4} \right) \Gamma \left(1 - \frac{K}{2} \right) \Gamma \left(\frac{1}{K} \right) \left\{ \psi' \left(\frac{K}{4} \right) - \psi' \left(1 - \frac{K}{4} \right) \right\}}{2^{2/K} \Gamma^2 \left(1 - \frac{K}{4} \right) \Gamma \left(\frac{K}{2} \right) \Gamma \left(\frac{1}{K} + \frac{1}{2} \right)} \quad (15)$$

[$\Gamma(z)$, $\psi'(z)$ are the Gamma and Trigamma functions, respectively]. The dependence of ν on K , the parameter which tunes the SIT in the SC wires, is depicted in Fig. 2 for different values of $T \ll v/a$. In this regime, ν exhibits a pronounced maximum at $K^*(T)$, slightly below the transition from SC to insulator ($K_c = 2$ [3, 20]). As T is lowered, the peak becomes progressively enhanced and $K^* \sim \sqrt{2}$ as dictated by the rightmost exponential factor in Eq. (14). This non-monotonous behavior can be traced back to the competition between electric resistance (which signifies the rate of phase-slips), and α_{xy} (which signifies the strength of diamagnetic response).

We next investigate the relation of α_{xy} to the diamagnetic magnetization density $M = -(1/LW)(\partial F / \partial B)$, assuming that the B -dependence of the free energy F is restricted to the flux in the junction area, i.e. the parameter q in Eq. (3). To leading order in H_J and at low

magnetic field $B = (\Phi_0/2\pi W)q$,

$$M \cong \frac{2\pi}{\Phi_0} \frac{T}{2L} \partial_q \langle S_J^2 \rangle_0, \quad (16)$$

$$S_J = -J \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \int_0^{\frac{1}{\tau}} d\tau \cos \{ \phi_1(x, \tau) - \phi_2(x, \tau) - qx \}.$$

The resulting expression for M is identical to Eq. (10) up to a constant prefactor. We thus obtain the relation

$$\alpha_{xy} = -\frac{M}{T_0}, \quad T_0 \equiv \frac{v}{\pi a} = \sqrt{E_c E_J} \quad (17)$$

where the last equality associates the energy scale T_0 with the plasma energy in the Josephson chain forming the legs of the ladder. This resembles the linkage pointed out in earlier literature, except the thermal energy scale T in the prefactor is replaced here by the characteristic scale of quantum dynamical phase-slips.

We finally note that the remarkable relation to the entropy per carrier $\alpha_{xy} \sim -(s/B)$, derived for clean systems [13, 14], does not hold here. Indeed, this relation can be recovered, e.g., employing the Boltzmann equation for an ordinary conductor in the limit $\omega_c \tau \rightarrow \infty$ (with ω_c the cyclotron frequency and τ a mean free time [21]). In contrast, for $\omega_c \tau \ll 1$ the same calculation yields $\alpha_{xy} \sim B\tau^2$. In our case, the latter limit is appropriate: while translational invariance holds in the x -direction, charge conductance along the y -direction is governed by weak tunneling between two discrete points, $\sim J$. We hence expect $\alpha_{xy} \sim BJ^2$, in accord with Eq. (10).

To summarize, we studied the transverse thermoelectric coefficients due to quantum SC fluctuations in a Josephson two-leg ladder, and their relation to diamagnetism. Most importantly, we predict a large Nernst signal, particularly at low T where a pronounced peak is predicted close to the SIT. This behavior reflects a subtle interplay between diamagnetism (favored in the SC phase), and dynamical phase-fluctuations (which proliferate in the insulator). As a concluding remark, we expect a qualitatively similar effect to hold in 2D SC films (or an infinite stack of such ladders). Possibly, it can also explain some properties of the existing data [6].

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